Practice Worksheet: Solving Linear and Non-Linear Systems of Equations

Objective:

- To practice solving linear and non-linear systems of equations.
- To apply numerical methods for solving both types of systems (i.e. Gradient descent, and Newton's method) using Python.

Part 1: Solving Linear System of Equations

Question 1:

Consider the following system of linear equations:

$$\begin{aligned} x + 2y - z &= 1 \\ 2x - y + 3z &= 5 \\ 3x + y + 2z &= 7 \end{aligned}$$

1. Using gradient-descent algorithm, solve this system of equations to find the unique solution for x, y, and z.
2. Verify that the solution satisfies all three equations.

Question 2:

Consider the following linear system of equations:

$$\begin{aligned} x - 3y + 4z &= 6 \\ 2x + y - z &= 3 \\ x + 2y + 3z &= 4 \end{aligned}$$

1. Using gradient-descent algorithm, solve this system of equations to find the unique solution for x, y, and z.
2. Verify that the solution satisfies all three equations.

Part 2: Solving Non-Linear Systems of Equations

Question 3:

Consider the following non-linear system of equations:

$$\begin{aligned} x^2 + y^2 - 4 &= 0 \quad \text{(Equation 1)} \\ x^3 - y &= 0 \quad \text{(Equation 2)} \end{aligned}$$

1. Solve this system using the **Newton-Raphson method** and **Gradient Descent**.
2. Plot the level curves of the equations and visualize the path of the solution.

Part 3: Mixed Linear and Non-Linear Systems

Question 4:

Solve the following system that consists of both linear and non-linear equations:

$$\begin{aligned} 2x + 3y - z &= 4 \quad \text{(Linear Equation 1)} \\ x^2 + y^2 - 4 &= 0 \quad \text{(Non-linear Equation 2)} \\ x^3 - y &= 0 \quad \text{(Non-linear Equation 3)} \end{aligned}$$

1. Use **Newton-Raphson Method** and **Gradient Descent** to solve this mixed system.
2. Visualize the solution and check if it satisfies the system.